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Applied Thermodynamics

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Adapted from: Y.A. Cengel, M. A. Boles, Thermodynamics – An Engineering Approach (5th Edition)

Chapter 4

Energy Analysis of Closed Systems

Introduction

$$E_{in} - E_{out} = \underbrace{(Q_{in} - Q_{out})}_{\text{= 0 if adiabatic}} + \underbrace{(W_{in} - W_{out})}_{\text{= 0 if no work transfer}} + \underbrace{\cancel{(E_{mass,in} - E_{mass,out})}}_{\text{= 0 if closed system}} = \Delta E_{system}$$

$$W = W_b + W_{other}$$

Introduction

$$E_{in} - E_{out} = Q_{net,in} - W_{net,out} = \Delta E_{system}$$

Convention from the reference book:

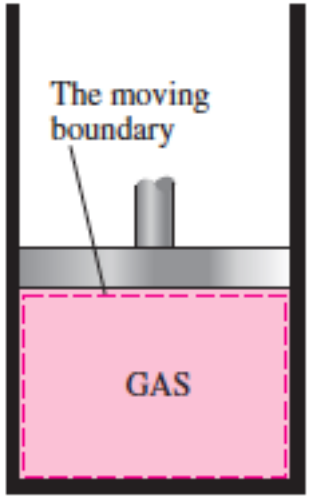
- *heat is transferred into the system (heat input) in the amount of Q*
- *work is done by the system (work output) in the amount of W*

In other words:

- *Q>0 if the heat is provided to the system*
- *W>0 is work is provided by the system*

Moving boundary work

- In a piston-cylinder devide, during the expansion or the compression, part of the boundary (the inner face of the piston) moves back and forth. Therefore, the expansion and compression work is often called moving boundary work
- Example: work produced by an internal combustion engine
- In the case of a quasi-static process, we can compute the path of the transformation and thus the work (which is maximum in that case).



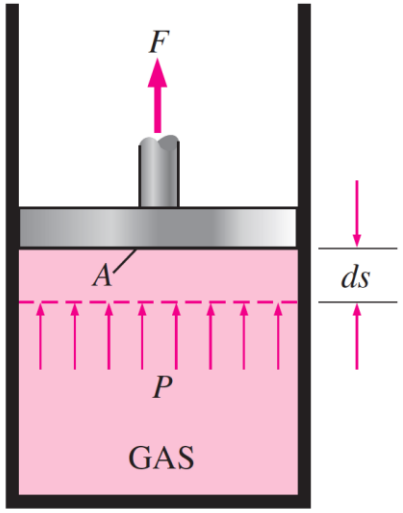
Moving boundary work

- A quantity of gas is enclosed in a piston-cylinder system
- Initial state: pressure P, Volume V
- Cross-sectional area of the piston: A
- If the piston is allowed to move a distance ds in a quasi-equilibrium manner, the differential work is given by:

$$\delta W_b = F.ds = P.A.ds = P.dV$$

- The work over the complete evolution is given by:

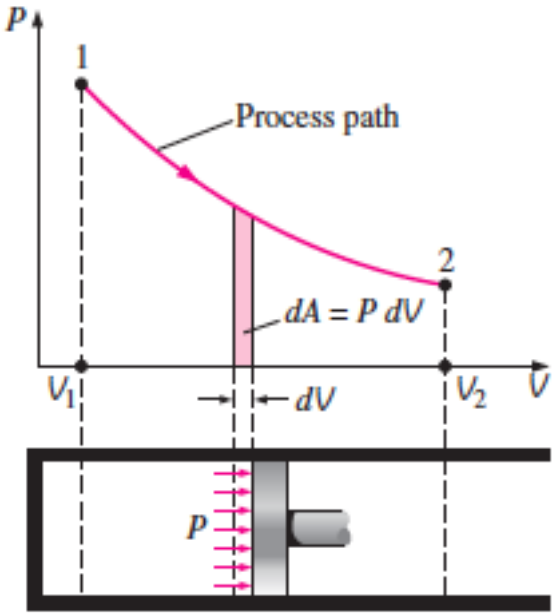
$$W_b = \int_1^2 P.dV$$

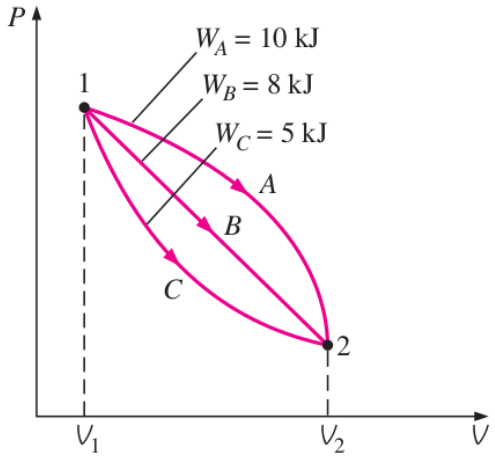


Moving boundary work

- This integral can be evaluated only if we know the functional relationship between P and V during the process.
- The area under the process curve on a P-V diagram is equal, in magnitude, to the work done during a quasi-equilibrium expansion or compression process of a closed system.

$$Area = A = \int_1^2 dA = \int_1^2 PdV$$





Moving boundary work

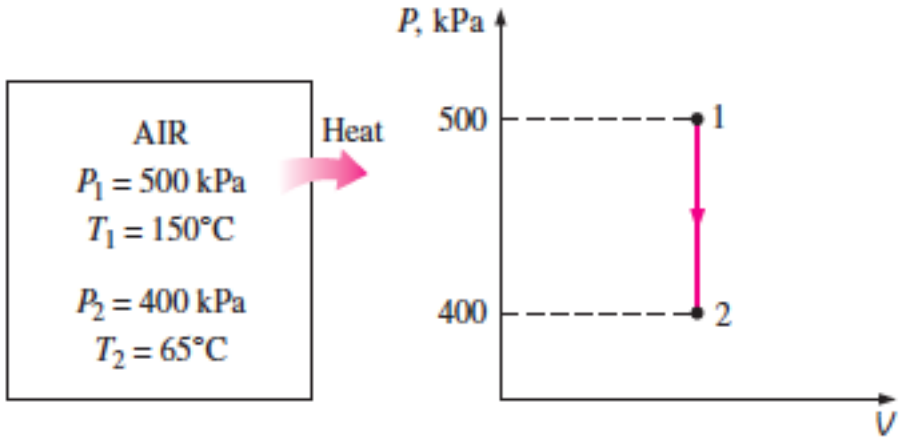
Constant volume process

Example 4-1:

- ✧ Rigid tank with air at 500 kPa and 150°C
- ✧ Cooling down the the following state: 400 kPa, 65°C
- ✧ What is the moving boundary work in this case?

$$W_b = \int_1^2 P dV = 0$$

The tank is rigid => dV=0
Work is zero



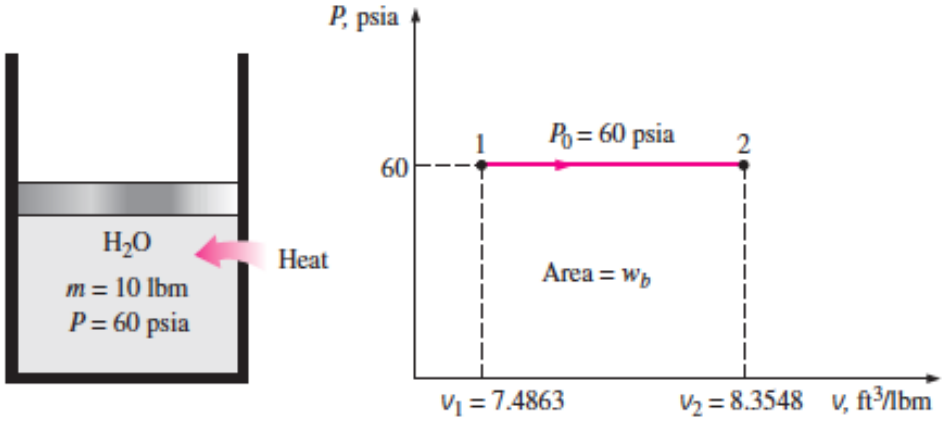
Moving boundary work

Constant pressure process

Example 4-2:

- ✧ Initially: vapor at 60 psia and 320°F.
- ✧ Heating up at constant pressure to reach 400°F.
- ✧ The piston is not connected to a shaft (free expansion).
- ✧ What is the work produced by the system?

$$W_b = \int_1^2 PdV = P_0 \int_1^2 dV = m.P_0.(v_2 - v_1)$$



Moving boundary work

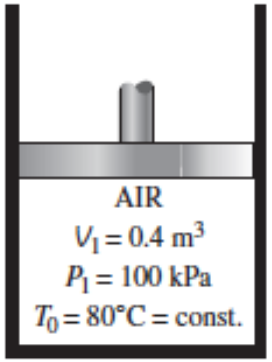
Isothermal process (case of an ideal gas)

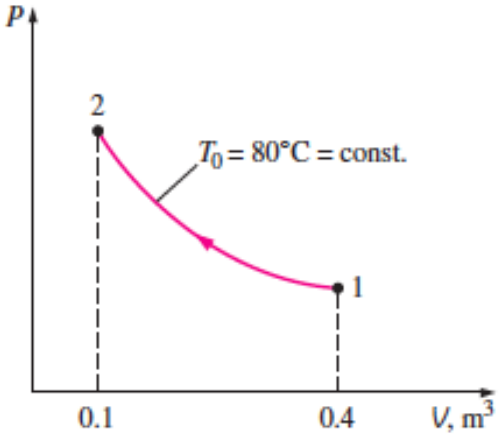
Example 4-3:

- ✧ Initially: 0.4 m^3 air at 100 kPa and 80°C
- ✧ The volume is reduced down to 0.1 m^3 while maintaining the air temperature constant
- ✧ What is the work produced by the system?

$$PV = mRT_0 = C$$

$$W_b = \int_1^2 PdV = \int_1^2 \frac{C}{V} dV = C \ln \frac{V_2}{V_1} = P_1 V_1 . \ln \frac{V_2}{V_1} = -55.5\ kJ$$





Energy balance of closed systems

Reminder:

For closed systems:

$$E_{in} - E_{out} = \underbrace{(Q_{in} - Q_{out})}_{\text{= 0 if adiabatic}} + \underbrace{(W_{in} - W_{out})}_{\text{= 0 if no work transfer}} + \underbrace{\cancel{(E_{mass,in} - E_{mass,out})}}_{\text{= 0 if closed system}} = \Delta E_{system}$$

$$\dot{E}_{in} - \dot{E}_{out} = \underbrace{(\dot{Q}_{in} - \dot{Q}_{out})}_{\text{= 0 if adiabatic}} + \underbrace{(\dot{W}_{in} - \dot{W}_{out})}_{\text{= 0 if no work transfer}} + \underbrace{\cancel{(\dot{E}_{mass,in} - \dot{E}_{mass,out})}}_{\text{= 0 if closed system}} = \frac{dE_{system}}{dt}$$

Energy balance of closed systems

- The variation of total energy is equal the variation of the internal energy plus the variation of kinetic and potential energy

$$\Delta E = \Delta U + \Delta KE + \Delta PE$$

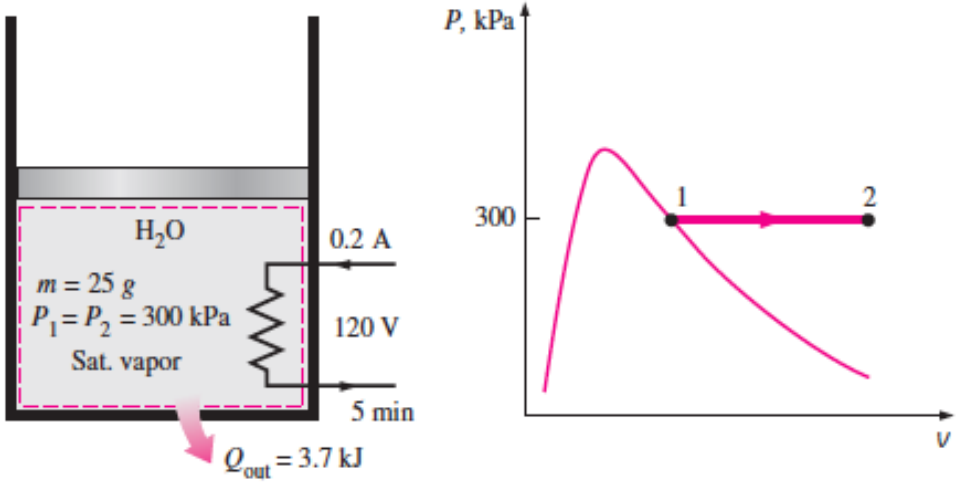
$$\Delta U = m(u_2 - u_1)$$
$$\Delta KE = \frac{1}{2} m(V_2^2 - V_1^2)$$
$$\Delta PE = mg(z_2 - z_1)$$

Energy balance of closed systems

Electric heating at constant pressure

Example 4.5:

- 25 g of saturated vapor at a constant pressure of 300 kPa
- A resistance heater within the cylinder is turned on and passes a current of 0.2 A for 5 min from a 120-V source.
- At the same time, a heat loss of 3.7 kJ occurs.
- *Determine the final temperature of the steam.*



Energy balance of closed systems

Electric heating at constant pressure

Solution:

- System: the cylinder, including electric resistances.
- Closed systems (no mass transfer)
- Kinetic and potential energy changes are null => E = U. The energy balance is written:

$$Q - W = \Delta U$$
$$\Leftrightarrow Q - W_{other} - W_b = U_2 - U_1$$

- The moving boundary work is:

$$W_b = P_0(V_2 - V_1) = P_2V_2 - P_1V_1$$

- And the energy balance becomes:

$$Q - W_{other} = (U_2 + P_2V_2) - (U_1 + P_1V_1) \underbrace{= H_2 - H_1}$$

The boundary work is directly accounted for by using the enthalpy!

Energy balance of closed systems

Electric heating at constant pressure

Solution:

- The only other form of work is electrical work:

$$W_{e,in} = 7.2\ kJ$$

- Enthalpy at the state 2 can then be calculated by:

$$-Q_{out} + W_{e,in} = m(h_2 - h_1)$$

With:

$$Q_{out} = 3.7\ kJ$$

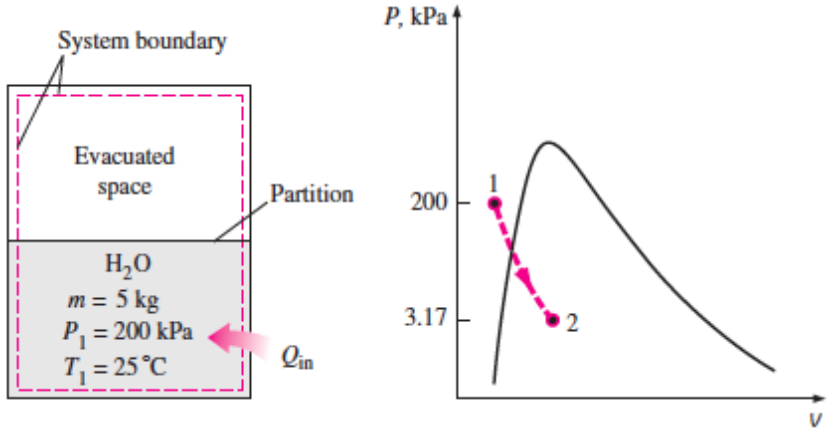
- Enthalpy and pressure are known for state 2 => T_2 can be calculated (state postulate)

Energy balance of closed systems

Unrestrained Expansion of Water

Example 4-6:

- A rigid tank is divided into two equal parts by a partition.
- Initially, one side of the tank contains 5 kg of water at 200 kPa and 25°C, and the other side is evacuated
- The partition is then removed, and the water expands into the entire tank.
- The water is allowed to exchange heat with its surroundings until the temperature in the tank returns to the initial value of 25°C.
- Determine (a) the volume of the tank, (b) the final pressure, and (c) the heat transfer for this process.



Energy balance of closed systems

Unrestrained Expansion of Water

Solution

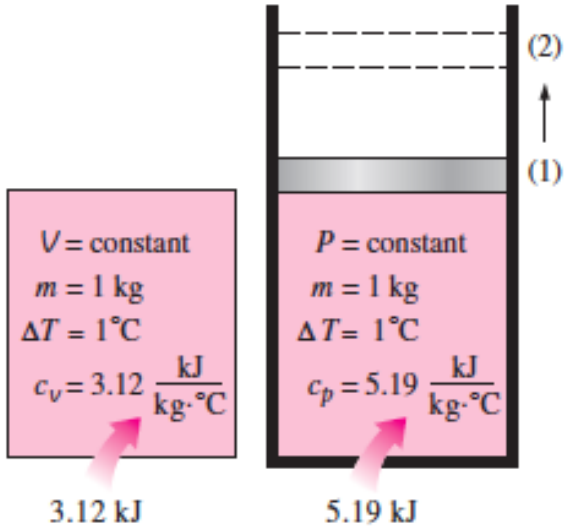
- Kinetic and potential energy changes are null
- Rigid tank => no boundary work
- Considered system: the inside of the tank
- Closed system (no mass exchange)

$$Q_{in} = \Delta U = m(u_2 - u_1)$$

Specific heat

- We know from experience that it takes different amounts of energy to raise the temperature of identical masses of different substances by one degree.
  - ex: 4.5 kJ to raise the temperature of 1 kg of iron from 20 to 30°C

    41.8 kJ to raise the temperature of 1 kg of water from 20 to 30°C
- it is desirable to have a property that will enable us to compare the energy storage capabilities of various substances.
- The **specific heat** is defined as the energy required to raise the temperature of a unit mass of a substance by one
- two kinds of specific heats:
  - At constat volume: c_v
  - At constant pressure: c_p



Specific heat

- Let's express the specific heats in terms of other thermodynamic properties.
- The conservation of energy for a constant-volume process system can be written in its differential form:

$$\delta e_{in} - \delta e_{out} = du$$

- The left side of the equation is the energy transferred to the system at constant volume. According to the definition c_v, it is equal to $c_v\, dT$
- And thus:

$$c_v = \left(\frac{\partial u}{\partial T}\right)_v$$

- Similarly:

$$c_p = \left(\frac{\partial h}{\partial T}\right)_P$$

Specific heat

- c_p can be defined as the change in the enthalpy of a substance per unit change in temperature at constant pressure.
- c_v can be defined as the change in the internal energy of a substance per unit change in temperature at constant volume

Internal Energy, Enthalpy and Specific heat

Case of ideal gases

- Reminder: an ideal gas is a gas whose temperature, pressure, and specific volume are related by:

$$Pv = rT$$

- It has been demonstrated mathematically and experimentally (Joule, 1843) that for an ideal gas the internal energy is a function of the temperature only:

$$u = u(T)$$

- By combining the two previous equations, we can show that the enthalpy of an ideal gas is a function of the temperature only:

$$h = u + Pv = u + rT$$

Internal Energy, Enthalpy and Specific heat

Case of ideal gases

- For an ideal gas, internal energy and enthalpy only depend on the temperature => specific heat at constant volume and pressure only depend on the temperature

➢ Partial derivatives become ordinary derivatives:

$$du = c_v(T)dT \quad (kJ/kg)$$
$$dh = c_p(T)dT \quad (kJ/kg)$$

➢ The change in internal energy or enthalpy for an ideal gas during a process from state 1 to state 2 is determined by integrating these equations:

$$\Delta u = u_2 - u_1 = \int_1^2 c_v(T)dT$$

$$\Delta h = h_2 - h_1 = \int_1^2 c_p(T)dT$$

?

Internal Energy, Enthalpy and Specific heat

Case of ideal gases

- To carry out these integrations, we need to have relations for cv and cp as functions of temperature.
- Polynomial equations (3rd order) are available in the literature to express the specific heat as a function of temperature
- u and h can also be calculated using tables

$$\Delta u = u_2 - u_1 = \int_1^2 c_v(T)\,dT$$

$$\Delta h = h_2 - h_1 = \int_1^2 c_p(T)\,dT$$

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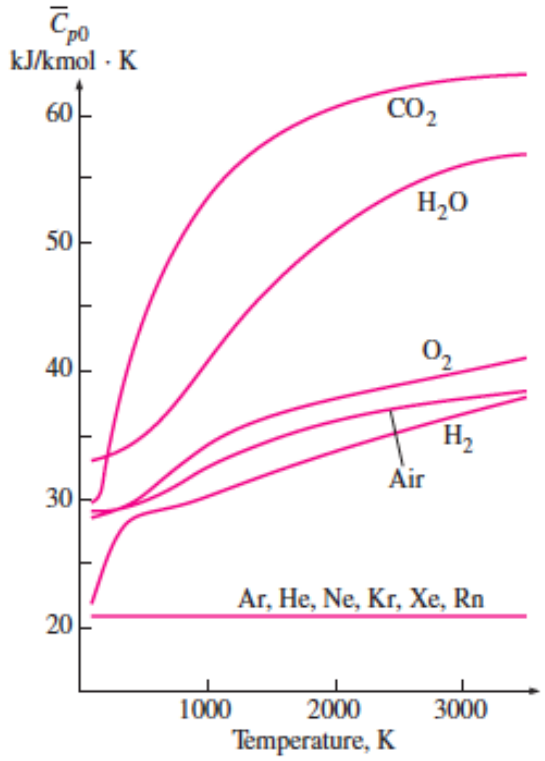
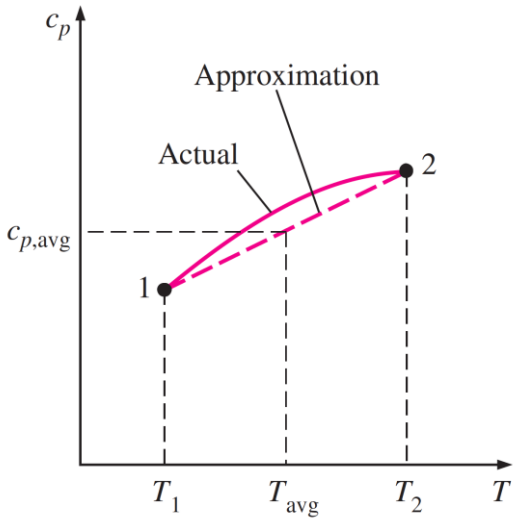


Internal Energy, Enthalpy and Specific heat

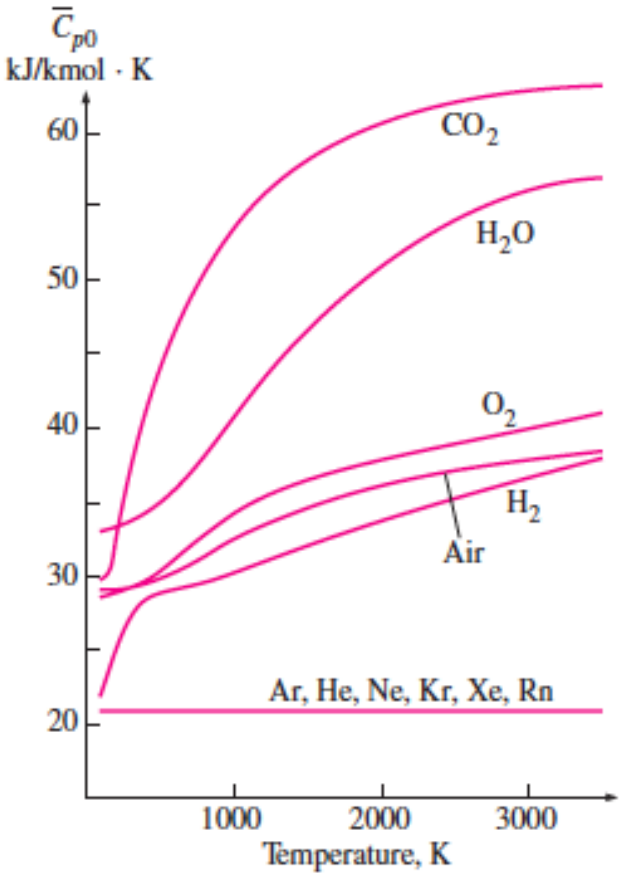
Case of ideal gases

NB:

- specific heats of monatomic gases such as argon, neon, and helium remain constant over the entire temperature range.
- Complex molecules have higher specific heat (non-constant)
- If c_p (or c_v) varie almos linearly with the temperature, an average value can be used



$$u_2 - u_1 = c_{v,avg}(T_2 - T_1) \quad (kJ/kg)$$



Internal Energy, Enthalpy and Specific heat

Case of ideal gases

- Specific Heat Relations of Ideal Gases:
  - ✧ Relation between c_p and c_v:

$$h = u + RT$$
$$\Leftrightarrow dh = du + RdT$$
$$\Leftrightarrow c_P = c_v + R \quad (kJ/kg\ K)$$

  - ✧ Important ideal-gas property: the specific heat ratio (1.4 for air):

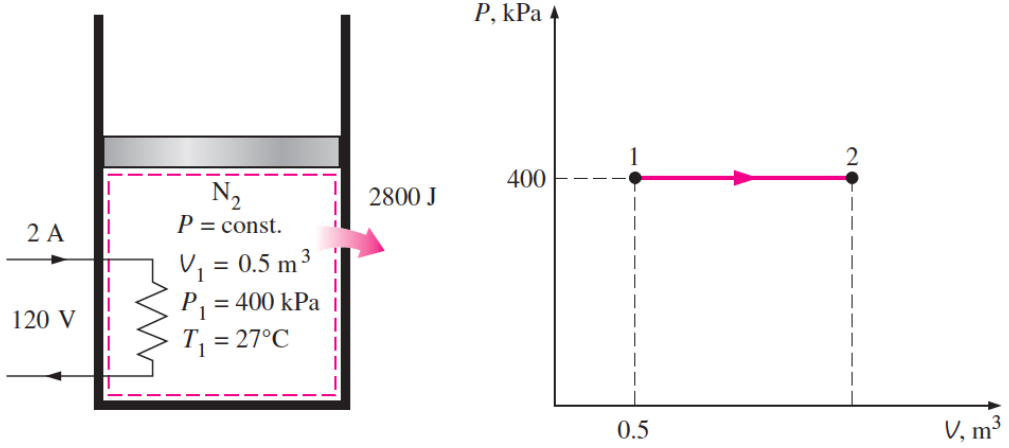
$$k = \frac{c_P}{c_v}$$

Internal Energy, Enthalpy and Specific heat

Case of ideal gases

Example 4-9:

- Piston-cylinder with 0.5 m3 nitrogen at 400 kPa and 27°C
- An electrical resistance is activated with 2 A and120V during 5 minutes.
- Nitrogen is expanded at constant pressure.
- Ambient heat losses are 2800 J.
- *Determine the final temperature of nitrogen*.



Internal Energy, Enthalpy and Specific heat

Case of ideal gases

Solution:

- Determine the mass of nitrogen using the ideal gas equation of state.
- Use the energy balance, which is written:

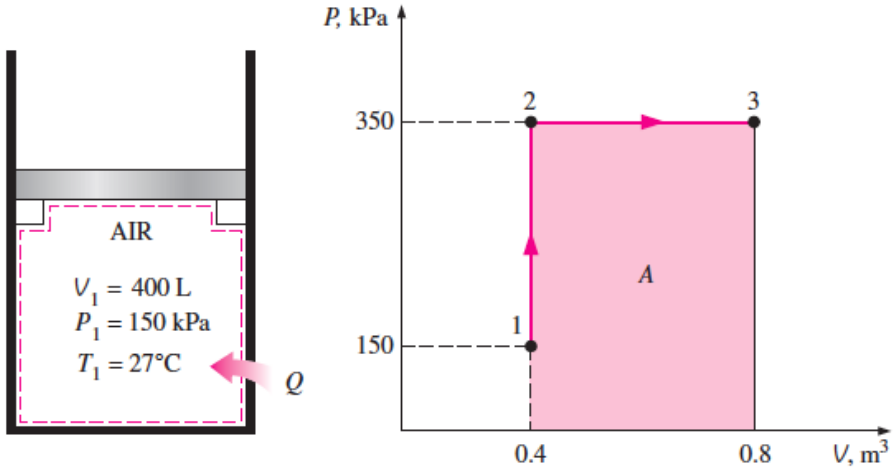
$$W_{e,in} - Q_{out} - W_{b,out} = dU$$
$$\Leftrightarrow W_{e,in} - Q_{out} = dH = m\, c_P\, (T_2 - T_1)$$

Internal Energy, Enthalpy and Specific heat

Case of ideal gases

Example 4-10:

- Piston cylinder with air at 150kPa and 27°C.
- Initially, the piston is resting on a pair of stops and the gas volume is 400 liters
- The mass of the piston is such that a 350-kPa pressure is required to move it
- The air is now heated (heat quantity: Q) until its volume has doubled
- Determine:
  - The final air temperature
  - The work produced by the system
  - The total heat provided to the air.



Internal Energy, Enthalpy and Specific heat

Case of ideal gases

Solution:

- The final air temperature is calculated by: $$\frac{P_1 V_1}{T_1} = \frac{P_3 V_3}{T_3}$$

- The work produced by the system is given by:

$$W_{13} = (V_3 - V_2).P_2$$

- The total heat transfer is calculated by:

$$E_{in} - E_{out} = \Delta E_{system}$$
$$\Leftrightarrow Q_{in} - W_{b,out} = \Delta U = m(u_3 - u_1)$$

- If the specific heat is assumed to be constant, we can use:

$$(u_3 - u_1) = c_v(T_3 - T_1)$$

Internal Energy, Enthalpy and Specific heat

Case of solids and liquids

- An incompressible substance is a substance whose specific volume (or density) is constant)
- Liquids and solids can be assumed as incompressible in good approximation
- Since they are incompressible, their specific heats at constant volume and pressure are identical:
- As a consequance, we can write:

$$c_P = c_v = c$$

- The specific heat only depends on the temperature:

$$du = c(T)dT$$

- The variation of internal energy can thus be written::

$$\Delta u = u_2 - u_1 = \int_1^2 c \quad (T)dT$$

Internal Energy, Enthalpy and Specific heat

Case of solids and liquids

- To compute the enthalpy, we can write it in its differential form:

$$dh = du + vdP + Pdv = du + vdP$$

- Integrating, we have:

$$\Delta h = \Delta u + v\Delta P \cong c_{avg}\Delta T + v\Delta P \quad (kJ/kg)$$

  - For solids, the term *vΔP* is negligible
  - For liquids if the transformation is performed at constant temperature (in a pump for example):

$$\Delta h = v\Delta P \quad (kJ/kg)$$

Which justifies the calculation of enthalpy for compressed liquids provided in chapter 3:

$$h_{@P,T} \cong h_{f@T} + v_{f@T}.\left(P - P_{sat,T}\right)$$

Exercise in EES

4–72 A piston–cylinder device, whose piston is resting on a set of stops, initially contains 3 kg of air at 200 kPa and 27°C. The mass of the piston is such that a pressure of 400 kPa is required to move it. Heat is now transferred to the air until its volume doubles. Determine the work done by the air and the total heat transferred to the air during this process. Also show the process on a P-v diagram. *Answers:* 516 kJ, 2674 kJ